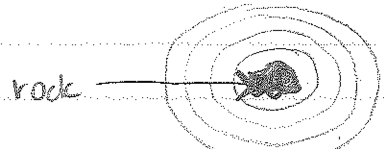
Chapter 7a - Day 1

Q1 - Attendance

For the next two class periods, will be discussing related rates problems. This means we'll be computing the rate of change of one quantity in terms of another.

This means we'll be taking derivatives with respect to different variables.

Ex: Consider dropping a rock in a pond and it creates ripples



The area of the outer circle is $A = \pi r^2$

... but the Area and radius are changing with time.

So if we take the derivative, we'd take it with respect to t (time)

Recall: Chain Rule: if y is a function of u and u is a function of x, then y is also a function of x and

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Ex: The area of a circle $A = \pi r^2$ and r depends on t. find a formula for $\frac{dA}{dt}$.

$\longrightarrow$

$$\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt}$$

Q2 now $\frac{dA}{dr} = 2\pi r$ using power rule

So then $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

Related Rates Problem Guidelines

1. READ the problem!
2. identify the variables and draw a picture
3. Write down what you know and what you are supposed to find.
4. find a relationship (formula) relating your variables
5. Use the chain rule to take a derivative
6. Plug in what you know and solve.

Ex: Boyle's Law states that when a gas is compressed at a constant temperature, the pressure P and volume V satisfy the equation PV=C where C is a constant. If at a certain instant the volume is 400 cm^3, pressure is 200 kPa, and the pressure is increasing at a rate of 25 kPa/min At what rate is the volume decreasing at this instant?

$$PV = C$$

take the derivative with respect to time

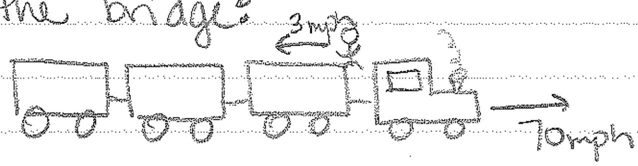
$$\frac{dP}{dt} \cdot V + P\frac{dV}{dt} = 0$$

$$(25)(400) + (200)\frac{dV}{dt} = 0$$

$$200\frac{dV}{dt} = -10{,}000$$

So $\frac{dV}{dt} = \boxed{-50}$ cm^3/min

Ex: A train is traveling across a bridge at 70 mph. A man on the train is walking toward the rear of the train at 3 mph. How fast is the man traveling across the bridge?

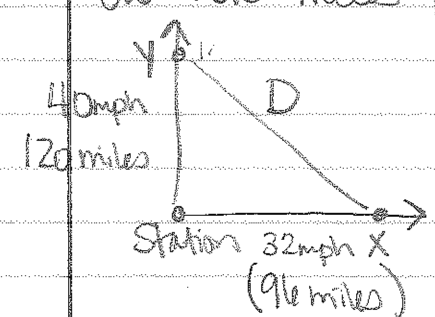


Subtract!

$70 - 3 = \boxed{67 \text{ mph}}$

Ex: 2 trains leave a station at the same time. One travels north on a track at 40mph. The second travels east on a track at 32 mph. How fast are they traveling away from each other in miles per hour when the northbound train is 120 miles from the station?

Q3 find t



$d = rt$

$120 = 40t$

$3 = t$

now find X

so $d = rt$

$d = 32(3) = 96$

*find D

$D^2 = 96^2 + 40^2 = 10{,}816$

$D^2 = x^2 + y^2$

so $D = \sqrt{10816} = 104$

take the derivative with respect to time

$$2D\frac{dD}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

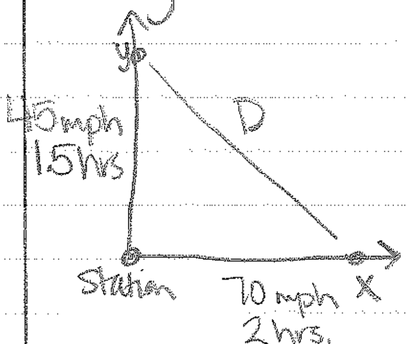
$$2(104)\frac{dD}{dt} = 2(96)(32) + 2(120)(40)$$

$$208\frac{dD}{dt} = 15{,}744$$

$$\frac{dD}{dt} = \frac{15{,}744}{208} = \boxed{75.692 \text{ mph}}$$

ran out of
time to cover
this problem!

Ex: 2 trains leave a station at noon. A northbound train travels at 45 mph, the eastbound train travels at 70 mph. At 1pm, the northbound train stops for ½ hour while the eastbound train continues at 70 mph without stopping. At 1:30, the northbound train continues north at 45 mph. How fast are the trains traveling away from one another at 2pm?



$\boxed{Q4}$ find x, y

again will use $x^2+y^2=D^2$
so we'll need x, y, D

$x=(70)(2)=140$

$y=(45)(1.5)=67.5$

then $D^2=140^2+(67.5)^2=24156.25$

$D=\sqrt{24156.25}$

now differentiate $D^2=x^2+y^2$ w/ respect to time

$$2D\frac{dD}{dt}=2x\frac{dx}{dt}+2y\frac{dy}{dt}$$

$$2(\sqrt{24156.25})\frac{dD}{dt}=2(140)(70)+2(67.5)(45)$$

$$2\sqrt{24156.25}\,\frac{dD}{dt}=25{,}675$$

$$\frac{dD}{dt}=\frac{25675}{2\sqrt{24156.25}}=\boxed{82.597\text{ mph}}$$